Scientific Electronic Archives
Issue ID: Sci. Elec. Arch. Vol. 17 (5)
Sept/Oct 2024
DOI: http://dx.doi.org/10.36560/17520241973
Article link: https://sea.ufr.edu.br/SEA/article/view/1973

ISSN 2316-9281

The contribution of working memory and spatial perception to the ability to solve geometric problems

Victoria Galitskaya
Net Media Lab Mind - Brain R&D IIT - N.C.S.R. "Demokritos"
Department of Primary Education, National and Kapodistrian University of Athens

Athanasios Drigas
Net Media Lab Mind - Brain R&D IIT - N.C.S.R. "Demokritos"

Alexandros-Stamatios Antoniou
Department of Primary Education, National and Kapodistrian University of Athens

Abstract. Geometry is a branch of mathematics that deals with the properties of space, including distance, shape, size, and the relative position of figures. It is one of the oldest branches of mathematics and has applications in various fields such as science, art, architecture, and even in areas seemingly unrelated to mathematics. Studies show that working memory and spatial perception contribute to students' geometry performance. This paper presents multiple studies demonstrating the brain regions activated when solving geometric problems. Interestingly, the brain areas activated when solving algebraic problems are different from those activated when solving geometric problems. Finally, multiple studies are presented that indicate students with learning difficulties lag in geometry, as solving geometric problems requires good reading and arithmetic skills.
Keywords: Brain Functions, geometry, recognition, ageometria, Learning Difficulties, Working memory

Introduction

Geometry is a branch of mathematics that deals with studying shapes, sizes, and properties of space. It contributes significantly to the understanding of the environment and is the basis for many fields such as architecture, engineering, and physics.

Geometry is taught to students from an early age, but teachers and research suggest that many students struggle with the subject (de Oliveira & Carneiro, 2022; Jumadi et al., 2022; Juman et al., 2022a; Segerby, 2023). Van Hiele proposed a theory of geometric reasoning that comprised five levels. Based on their geometric ability, students were categorized into the corresponding levels. Several scholars conducted research to examine the validity of the theory, determine the characteristics of each level, assess the student's level, and implement teaching strategies based on the theory (Anđelković &Malinović-Jovanović, 2023; Arnal-Bailera& Manero, 2023; Bossé et al., 2021; Celik & Yilmaz, 2022; González et al., 2022; Hassan et al., 2020b, 2020a; Kusuma et al., 2021; Lumbre et al., 2023; Mahlaba &Mudaly, 2022; Moru et al., 2021a; Naufal et al., 2020a, 2020b, 2021b, 2021a; Pujawan et al., 2020; Roldán-Zafra et al., 2022; Utomo et al., 2023a; UYGUN & GÜNER, 2021; Wahyuni et al., 2020; Zhou et al., 2022). The results were not so good as it turned out that the students' geometric abilities were not so well developed. However, when the teachers applied Van Hiele's theory in teaching geometry it was found that the geometric abilities of the students improved (Armah & Kissi, 2019; Machisi& Feza, 2021; Moru et al., 2021b; Utomo et al., 2023b). ZalmanUsiskin based on the theory of Van Hiele constructed a geometric thinking classification test called "Van Hiele Geometry Test (VHGT)" (Usiskin& Senk, 1990). This classification tool has been translated and used in many researches in countries such as the United States, Turkey, China, Indonesia, Malaysia, the Philippines, Singapore, Greece, Hungary, the Netherlands, Portugal, Slovakia, the UK, Ghana, Israel, Palestine, South Africa, Brazil, and Canada. Moreover, it covers all levels of education as it has been applied to students attending elementary, middle school, high school, and university (Chen et al., 2023; Ma et al., 2015; Senk et al., 2022).

Research shows that students often struggle with geometry due to weakness:

1) understanding of the data and the issues involved,
2) application of theories (Galitskaya& Drigas, 2023; Juman et al., 2022b),
3) in working memory functions.

The visual-spatial deficits of working memory are shown to contribute to students' performance in geometry (Galitskaya& Drigas, 2023; Juman et al., 2022b; Linde-Domingo & Spitzer, 2022; Rivella et al., 2021) This is evidenced by a recent study conducted on 4th-grade students which highlighted the significant effect of visuospatial working memory on their ability to solve geometric problems such as calculating the perimeter and area of shapes (Xie et al., 2022). Working memory is essential not only for performing the processes necessary to solve problems (Passolunghi et al., 2015; Passolunghi& Mammarella, 2010, 2012; Caviola et al., 2012; Mammarella et al., 2013; Bull et al., 2008; Passolunghi et al., 2008) but also for temporarily retaining (Giofré et al., 2014) and storing information (e.g., verbal and audiovisual)(Angelopoulou et al., 2021b, 2021a). The capacity of working memory is limited, and it is needed to maintain processes and store information (Carretti et al., 2009; García-Madruga et al., 2013). This information stays in working memory for a short period, serving as an interface between perception, long-term memory, and action (Moser-Mercer, 2023).

In addition to working memory, spatial perception also plays a crucial role in the ability to solve geometric problems (Liapi, 2002).

Working memory

WM has been shown to predict success in school tasks that require information maintenance and processing, such as reading comprehension (Blankenship et al., 2015; García-Madruga et al., 2013; Siquara et al., 2018), approximate addition (Caviola et al., 2012a), multi-digit operations (Heathcote, 1994), representation size (Pelegrina et al., 2015), and mathematical achievement (Pappas et al., 2018; Passolunghi et al., 2008). Based on these, it seems reasonable to assume that WM is also involved in geometry learning.

According to recent reports, visuospatial WM may have a critical role in both arithmetic (Li & Geary, 2013; Szucs et al., 2013) and geometric processes (Mammarella et al., 2013). Geometry involves the processing of shapes in space and it seems plausible that, in addition to visuospatial WM, other visuospatial abilities also influence geometric learning (Hannafin et al., 2008). It was found that children who fail geometry differ from those who fail only arithmetic (Bizzaro et al., 2018a). Children with poor geometric learning have small deficits in calculation skills but struggle in other tasks, confirming that many factors are associated with deficits in geometric learning. Children with poor geometric learning perform poorly on almost all WM tasks, confirming that the verbal, visual, and spatial aspects of WM are involved in geometric learning.

Allen et al. (2020) conducted a study that aimed to identify the contribution of verbal, spatial-concurrent, and spatial-sequential measures of working memory to mathematical problems. The results revealed that the contribution of verbal working memory was greater than that of spatial working memory. Visual-spatial working memory is involved in students' performance in mathematics in a number of domains such as arithmetic(Caviola et al., 2012b), word problem-solving (Bühner et al., 2008; Swanson & Fung, 2016; Van de Weijer-Bergsma et al., 2015; Zavitsanou& Drigas, 2021; Zheng et al., 2011) and geometry (Giofrè et al., 2013a; Giofré et al., 2014), as well as mathematical difficulties (Allen et al., 2020; Galitskaya& Drigas, 2021; Szucs et al., 2013).

Spatial Perception

Spatial perception is another contributing factor to student success in geometry (Brown & Heywood, 2011; Foley et al., 2004; Hatfield, 1984; Wagner, 2008). It has been found that Homo erectus, to build Acheulean tools required advanced spatial cognitive abilities (Ferreirós& García-Pérez, 2020). In general, research has shown that adults, children, and animals are sensitive to the geometric properties of space and use these properties to identify objects and shapes or navigate the environment (Izard et al., 2011).

Spatial ability requires one to be able to perceive the horizontal-vertical, to be able to make mental rotations in space. There is strong evidence linking 3D spatial visualization to geometry compared to algebra (Newcombe et al., 2019). However, some other studies such as Kyttälä and Lehto show a direct relationship between visuospatial working memory (VSWM) and performance on algebraic word problems for high school students (Kyttälä& Lehto, 2008). However, they did not find a comparable relationship between VSWM and geometry problem-solving in this population. Even in cases where such a relationship was identified, the relationship between VSWM and geometry achievement appeared to be very weak and may have been limited only to tasks involving mental manipulation (Giofrè et al., 2013b).

Through meta-analyses, differences in visuospatial memory (VSWM) have been observed between the two sexes, with men having a small but significant advantage (Nordvik & Amponsah, 1998; Voyer et al., 2017). Age is an important moderator, showing an increase in the magnitude of gender differences with age (Voyer et al., 2017). However, the TIMSS report finds mixed gender comparison results, which list roughly equal results in boys and girls in geometry in elementary and middle school. Battista (1990) reported that males scored significantly higher than females in spatial visualization, geometry achievement, and geometric problem-solving. In contrast, other researchers have interpreted gender differences as a result of lower student self-efficacy (Fennema & Leder, 1990), stereotyping higher anxiety (Ganley & Vasilyeva, 2014), or a preference for using problem-solving strategies that have been learned or seen to work on similar problems (Bergstrom & Zhang, 2016; Gallagher & Kaufman, 2005).

VSWM predicts an individual's success in geometric activities (Giofrè et al., 2013b). Researchers argue that spatial perception is directly linked to geometric thinking(Owens &Outhred, 2006; Piaget, 1952). Maier claimed that spatial ability or spatial reasoning from five aspects (Maier, 1996):

1. Spatial visualization, is an individual's ability to make mental processes of objects in two or three dimensions.
2. Mental rotation, an individual's ability to rotate objects in two or three dimensions
3. Spatial orientation, is an individual's ability to understand the arrangement and position of elements within a shape.
4. Spatial perception, is an individual's ability to understand abstract spatial principles (horizontal-vertical).
5. Spatial relations, are an individual's ability to understand the spatial configuration of an object.

Evidiasari et al. (2019) conducted a study in which 35 secondary school students participated, who were divided into three groups so that they could be tested in spatial visualization, mental rotation, and spatial orientation, from each group the best-performing student was selected. The results of the study showed that the students belonging to the spatial visualization group used drawing and non-spatial strategies to solve geometric transformation problems. The student solved the spatial transformation problem by drawing auxiliary lines that connected each vertex of the object to its center. Students who belonged to the mental rotation group used holistic and analytical strategies in solving geometric transformation problems. Using a holistic strategy means imagining the whole transformation. Students belonging to the spatial orientation group did not require mental representations and students could only determine the position and orientation of an object in the solution (Evidiasari et al., 2019)

Hambrick found that subjects with low geospatial knowledge but high levels of spatial ability showed almost the same results as participants who showed high geospatial knowledge (Hambrick et al., 2012). Learning spatial relationships and properties of shapes is vital for students to succeed in science, technology, engineering, and mathematics (STEM) subjects at the college level (Hsi et al., 1997; Wai et al., 2010).

Brain functions and geometry

The relationship between brain functions and geometry is a fascinating area of research, shedding light on how the brain processes geometric information and its implications for education. Studies show distinct neural mechanisms involved in processing geometric versus algebraic concepts, with geometric terms eliciting greater activation in certain brain regions, such as the intraparietal sulcus At the brain level, the processing of information about direction (orientation and symmetry) appears to be separate from object recognition. For example, Turnbull describes a double dimension between two cases of brain-injured patients. A first patient, who suffered from a parietal lobe lesion, was able to name object images, but could not say when the images were presented in an unusual orientation. Conversely, a second patient could identify variations in orientation, but could not name the images. Thus, the brain structures responsible for object recognition appear to be removed along the orientation and sense properties of 2-dimensional shapes (Izard &Spelke, 2009).

Research has shown that geometric shapes found in the environment exert a strong control on reorientation behaviour, but the neural and cognitive mechanisms underlying this phenomenon are not well understood. While some theories claim that geometry controls behaviour through an allocentric mechanism possibly linked to the hippocampus, others argue that disoriented navigators achieve their goals using an egocentric projection-matching strategy.

The parietal cortex is involved in complex mathematical processes such as solving word problems, algebraic equations, and constructing geometric proofs (Leikin et al., 2014). Seeking different proofs of a particular theorem in geometry enhances reasoning and deductive reasoning (Hansen, 1998; Neubrand, 1998). Testing geometry requires greater cognitive control and activation of working memory, as shown by electrical activation in the anterior parts of the scalp (Arsalidou& Taylor, 2011; Newman et al., 2011). Intense activity in the posterior (particularly right) parts of the scalp during geometry testing may be linked to greater demands on visuospatial processing, including manipulation of internal representations in geometry problem-solving (Leikin et al., 2014; Zacks, 2008).

Leikin, Waisman, and Shaul, in 2014, published research in which they argued that algebraic tasks require translation from a graphical to a symbolic representation of a function, while tasks in geometry require translation from a drawing of a geometric measurement to a symbolic representation of its property. The findings showed that the electrical brain activity associated with performing geometric tasks is stronger than that associated with solving algebraic tasks. In addition, different scalp topographies of brain activity associated with algebraic and geometric tasks were found. Based on these results, they argued that problem-solving in algebra and geometry is associated with different patterns of brain activity. They noted that the differences in scalp topographies (expressed in interactions test laterality, test causality) revealed in this research at both visual and symbolic stages demonstrate that problem-solving in algebra and geometry is associated with the activation of different brain regions (Leikin et al., 2014).

In 2018, Marzia Bizzaroa, David Giofrèb, Luisa Girellia, and Cesare Cornoldic conducted a study that aimed to find the causes that contribute to students' failure in geometry. Fifth and sixth-grade students participated in the research. The students were divided into two groups, one group consisted of students who had ageometry and the other was the control group. The children were given problems (numerical and geometrical) which they had to solve. Based on the results, the researchers concluded that children with ageometry have a problem with working memory (verbal and visuospatial memory). A very important finding was that children who fail in geometry (children with ageometry) are different from those who only fail in arithmetic (children with dyscalculia)(Bizzaro et al., 2018b).

Similar studies comparing the processing of algebraic and geometric terms (for example, terms such as square and absolute value) revealed that geometric terms elicit greater activation in the intraparietal sulcus than algebraic terms (Zhang et al., 2012). Another study that investigated brain activity associated with the construction of geometric evidence revealed greater involvement of right hemispheric regions, particularly in the retrieval of geometric knowledge (Kao et al., 2008; Leikin et al., 2014).

Specific learning difficulties

Children with specific learning disabilities (SLD) are characterized by high scores on the Verbal Comprehension Index (VCI) and Perceptual Reasoning Index (PRI), and significantly lower scores on the working memory index (WMI) and processing speed index (PSI) (Poletti, 2016). They may have specific weaknesses in certain cognitive abilities that support academic learning, including working memory ability (Peng & Fuchs, 2016).

Many studies have documented WM deficits in children with different types of learning disabilities. These include reading difficulties (Gathercole et al., 2006), mathematical difficulties (Geary et al., 2007), and a combination of reading and mathematical difficulties (e.g.,(Swanson & Beebe-Frankenberger, 2004)). WM should be considered a general domain that contributes to learning (Baddeley, 1986) and deficits in working memory are associated with all types of learning disabilities (Peng & Fuchs, 2016). Children with learning disabilities have extensive deficits in WM and the severity of WM deficits varies by domain and type of learning disability (children with dyslexia lag in verbal WM, while students with dyscalculia lag in visual-spatial WM) (Swanson et al., 2009; Swanson & Jerman, 2006).

Peng and Fuchs considered studies conducted on children aged 5-20 years. The sample included students with reading difficulties (RD), children with mathematics difficulties (MD) children with comorbidity (RDMD) and children with typically developing (TD). The aim was to clarify whether children with RD and MD, or both together, have different deficits in working memory. They found that RD, MD and RDMD children showed similar WM deficits in both verbal and numerical domains. In addition, RD children appear to show comparable verbal and numerical WM deficits, and deficits in verbal WM appear the same as those of MD children. This suggests that RD children do not suffer from more severe verbal WM deficits than MD children and that the verbal WM deficit may represent a common feature of RD and MD children. In this model, the central processor is an important component and is responsible for coordinating the phonological loop and audiovisual sketch systems, directing attention to relevant information and suppressing irrelevant information and inappropriate actions (Baddeley, 1986), and it may be that the central processor problem is associated with, or a common cause of, LD learning disabilities (Peng & Fuchs, 2016).

Deficits in visual and spatial functional memory explain the difficulties of students with ageometria (Passolunghi& Mammarella, 2012). Visual working memory is a visual-spatial storage system and is an important predictor of students' performance in solving geometric problems. Unfortunately, many children with learning disabilities in mathematics show deficits in working memory (Zhang, 2017). Student experience and logical reasoning have been shown to play a critical role in the development of geometric skills, and there is no doubt that inappropriate geometry instruction (Clements, 2004) is a major reason for students' failure to learn geometry.

Research conducted in secondary education shows that students with learning disabilities in mathematics cannot represent, and develop strategies when solving mathematics word problems (Krawec, 2014; Van Garderen & Montague, 2003a). For problem-solving, problem representation is the first step in which students interpret a problem and process the data, and information to understand and then transfer the necessary information into a mathematical, visual or mental model to solve Problem representation strategies, for word problems, including schema activation, creating visual representations, and language paraphrases in text (Gonsalves & Krawec, 2014).

Zhang, Ding, Ding, and Mo argue that students with learning disabilities in mathematics often have difficulty solving geometry problems, which require strong visual representation skills. These difficulties have been associated with deficits in visual working memory. Students with learning disabilities often experience difficulties with the spatial representation of mathematical information and relationships and in manipulating visual-spatial relationships (Geary, 2003; Van Garderen & Montague, 2003b). In addition, students with mathematics disabilities fall short in geometry-related domains (Zhang et al., 2012).

Conclusion

The challenge in learning geometry is not just about its complexity. Factors like computational skills, working memory, and visual-spatial imagery also play a role. Working memory, for example, is crucial for retaining verbal and visual information needed in geometry. It predicts success in tasks like reading comprehension and mathematical achievement. Visuospatial working memory, especially, is important for geometry, which involves processing shapes in space. Children struggling with geometry often face difficulties in various working memory tasks. Research suggests that verbal working memory has a greater impact on mathematical problem-solving. Visual-spatial working memory affects performance in arithmetic, word problem-solving, and geometry, as well as in cases of mathematical difficulties.

Spatial perception is crucial for geometry success, with Homo erectus requiring it for tool-making. Both genders exhibit sensitivity to spatial properties. Visuospatial working memory (VSWM) predicts geometric activity success, although its link to geometry problem-solving varies among high school students. Gender differences in VSWM exist, with men having a slight advantage. Spatial ability, including visualization and mental rotation, is key for STEM success. Understanding spatial relationships is vital for college-level STEM subjects.

Research on the brain's role in geometry highlights distinct neural mechanisms for geometric versus algebraic concepts, with geometric terms activating specific brain regions like the intraparietal sulcus. Brain structures responsible for object recognition appear distinct from those for orientation processing. Understanding the brain's response to geometric shapes aids in reorientation behaviour studies. Parietal cortex involvement in mathematical processes includes solving word problems and constructing geometric proofs, with different brain activation patterns for geometry versus algebra. Geometry tasks engage visuospatial processing and working memory, as shown by scalp electrical activity. Studies reveal that geometry failures in students are linked to working memory issues. Geometric terms evoke greater brain activation than algebraic terms. Right hemispheric regions are heavily involved in geometric knowledge retrieval.

Children with specific learning disabilities (SLD) typically exhibit high scores in Verbal Comprehension and Perceptual Reasoning, but lower scores in Working Memory and Processing Speed. Working Memory (WM) deficits are common across various learning disabilities, including reading and mathematical difficulties, impacting academic learning. WM deficits vary by domain and type of disability; for instance, dyslexic children often lag in verbal WM, while those with dyscalculia lag in visual-spatial WM.

Research suggests that children with reading difficulties (RD), mathematical difficulties (MD), or both (RDMD) exhibit similar WM deficits. Deficits in WM, particularly verbal and numerical, are comparable between RD and MD children, indicating a common feature. Visual working memory, crucial for solving geometric problems, often faces deficits in children with mathematical learning disabilities. Geometry instruction inadequacies contribute to students' failure in learning geometry, with difficulties in problem representation and solving common among students with mathematical disabilities. Deficits in visual working memory hinder spatial representation and manipulation skills necessary for geometry problem-solving. Overall, addressing working memory deficits and providing effective geometry instruction are essential for supporting children with learning disabilities in mathematics.

Research demonstrates the effectiveness of using information and communication technology (ICT) in teaching mathematics, literature, and other subjects to both typically developing children and students with learning difficulties. Early intervention using ICT-based methods, such as educational apps, video games, and robotics, has shown promise in improving cognitive skills, and attention, for students with learning difficulties (Angelopoulou & Drigas, 2022; Brainin et al., 2021; Chaidi et al., 2021; Dorouka et al., 2020; Doulou et al., 2022; Drigas et al., 2014, 2020; Drigas & Politi-Georgousi, 2019; Galitskaya& Drigas, 2023; Kefalis et al., 2020; Sisman et al., 2021; Skiada et al., 2014; Stathopoulou et al., 2019, 2020). These approaches can be implemented as early as the kindergarten level to support the development of essential skills and address the challenges faced by this population (Drigas et al., 2015a, 2015b; G. Kokkalia et al., 2016; G. K. Kokkalia & Drigas, 2015).

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